

1.4.

- $P(0)$ true value.
- $P(4)$ true value.
- $P(6)$ false value.

5. a) there exist student who spends more than 5 hours every weekday in class.

b) All students spends more than 5 hours every weekday in class.

c) not all students spend more than 5 hours every weekday in class.

d) Every students do not spend more than 5 hours every weekday in class.

7. a) $\forall x (Cx) \rightarrow F(x)$.
all comedians is funny.

b) $\forall x (Cx) \wedge F(x)$
some co

c) $\exists x (Cx) \rightarrow F(x)$
Some comedian is funny.

11. a) $P(0)$. $0^2 = 0$. true.

b) $P(1)$ $1 = 1$ true.

c) $P(2)$ $2 \neq 2^2 = 4$ false

d) $P(-1)$ $-1 \neq (-1)^2 = 1$ false

e) $\exists x P(x)$. true.

f) $\forall x P(x)$ false.

17.

a). $\exists x P(x)$.

$P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$

b). $\forall x P(x)$.

$P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c). $\exists x \neg P(x)$.

$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d). $\forall x \neg P(x)$.

$\neg P(0) \vee \neg P(1) \vee \neg P(2)$

$\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e). $\neg \exists x P(x)$.

$\neg (P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

f). $\neg \forall x P(x)$

$\neg (P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

27. a). D: a students ~~is~~ in my school who live in Vietnam.

~~is~~ V: a student x.

$\exists x P(x)$.

D. a students live in school, students live in Vietnam.

V. a student x.

$\exists x (P(x) \wedge C(x))$.

$\exists x (A(x) \wedge P(x, \text{Vietnam}))$

b). $\exists x P(x)$.

D: students in my school who can't speak Hindi.

$\exists x (P(x) \wedge C(x))$.

D. students in my school.

People ~~students~~ can speak Hindi.

$\exists x (P(x) \wedge C(x, \text{Hindi}))$

D. students in my school.

students who ~~spea~~ speak Hindi.

c). $\exists x P(x)$.

D: people who know java, Prolog, and C++.

$\exists x (P(x) \wedge C(x))$

D. students who know java, Prolog and C++ who also live in school

30. a) $P(1,3), P(2,3), P(3,3)$
 $P(2,3)$.

b) $P(1,1) \wedge P(1,2) \wedge P(1,3)$

c) $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

d) $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

50. Suppose $\forall x P(x) \vee \forall x Q(x)$ is true.

① $[P(x_1) \wedge P(x_2) \wedge P(x_3) \dots P(x_n)]$ or
 $[Q(x_1) \wedge Q(x_2) \wedge Q(x_3) \dots Q(x_n)]$ is true.

② $\neg \forall x P(x) \wedge \neg \forall x Q(x) \rightarrow$ false.

④ $\exists x \neg P(x) \wedge \exists x \neg Q(x) \rightarrow$ false.

⑤ $[\neg P(x_1) \vee \neg P(x_2) \vee \neg P(x_3) \dots \neg P(x_n)]$ and

$[\neg Q(x_1) \vee \neg Q(x_2) \vee \neg Q(x_3) \dots \neg Q(x_n)] \rightarrow$ false.

⑥ $\neg (\exists x \neg P(x)) \vee \neg (\exists x \neg Q(x)) \rightarrow$ true.

$\neg (\exists x \neg P(x)) \vee \exists x \neg Q(x) \rightarrow$ true.

$(\exists x \neg P(x)) \vee \exists x \neg Q(x) \rightarrow$ false.

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1.5

1. a) for all x there exist at least a number y such that $y > x$.

b) For every number x and y ~~there~~ who is greater than 0, their product is greater or equal to 0.

c) There exist a number z , for all x and y , ~~same~~ x times y equal to z .

21. $\forall x \exists a \exists b \exists c \exists d ((x > 0) \rightarrow a^2 + b^2 + c^2 + d^2)$ $\forall x \exists z \exists y ((x, y, z))$

27. a) true.

b) true.

c) true.

d) true.

e) true.

f) false.

g) false.

h) true.

i) false.

30. a) a) $\forall x \forall y \neg P(x, y)$

b) $\exists x \exists y \neg P(x, y)$

~~c) $\forall y \neg (Q(y) \wedge \forall x \neg P(x, y))$~~

d) $\forall y \neg (\exists x P(x, y) \vee \forall x \neg P(x, y))$

e) $\forall y (\exists x \neg P(x, y) \wedge \forall x P(x, y))$

45 a) true.
b) false.
c) true.

~~48. $\forall x P(x) \wedge \forall y Q(y)$~~

48. $(\forall x P(x) \vee \forall x Q(x))$

$$\equiv [P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \dots P(x_n)] \vee [Q(x_1) \wedge Q(x_2) \wedge Q(x_3) \wedge \dots Q(x_n)]$$

$$\equiv \{P(x_1) \vee [Q(x_1) \wedge Q(x_2) \wedge Q(x_3) \wedge \dots]\} \wedge \{P(x_2) \vee [Q(x_1) \wedge \dots]\} \wedge \{P(x_3) \vee [Q(x_1) \wedge \dots]\} \wedge \dots$$

$$\equiv [P(x_1) \vee Q(x_1)] \wedge [P(x_2) \vee Q(x_2)] \wedge \dots [P(x_n) \vee Q(x_n)]$$

$$\equiv \forall x (P(x) \vee Q(x))$$

$$\equiv \forall x \forall y (P(x) \vee Q(y)).$$